PROVISIONING AN AIRCRAFT WEAPONS SYSTEM

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THESIS

PROVISIONING AN AIRCRAFT
WEAPONS SYSTEM

by

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September 1975

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by

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I. INTRODUCTION

This thesis examines the methodology involved in the selection of a spare parts inventory that supports a new aircraft weapons system. Chapter I reviews the background of the provisioning process, and chapter II examines the initial outfitting model that is used by the Aviation Supply Office (ASO). Chapter III deals with a sensitivity analysis of the aforementioned model, while chapter IV treats the possibility of using a more complex probability distribution in the inventory selection process. The last chapter summarizes the results of this investigation as well as recommendations.

A. DEFINITION AND STATEMENT OF PROBLEM

Provisioning is defined in reference 1 as:

> A management process to determine and acquire a range and a quantity of support items to maintain and to operate an end item for an initial period.

The provisioning process determines and procures support items, i.e., spare parts, test equipment, and ground support equipment for two echelons – the actual support site where the aircraft weapons system is operated and the supply system back-up or pipeline stocks that support the operational site. Provisioning for the first echelon only will be examined in this thesis.

It is apparent that financial constraints will always be an inevitable factor in inventory decisions. Therefore, the

basic problem in provisioning is the procurement of an optimal inventory that will support a weapons system effectively within a limited financial budget.

B. NAVY MAINTENANCE AND SUPPLY SUPPORT SYSTEMS

1. Maintenance Levels

There are three levels of maintenance in Naval Aviation. The most important is the Organizational Level (ORG) where the aircraft weapons system is operated. Limited maintenance is performed at this level, and the only spare parts that are usually permitted are of a pre-expended (in an accounting sense), consumable nature which would normally be used in a thirty-day period. The chief function of the organizational maintenance is to locate apparent failures in sub-systems.

The failed unit is returned to the supporting establishment which provides supply support and the second maintenance level support by the Intermediate Maintenance Activity (IMA).

If the failed unit is beyond the capability of the IMA, (BCM), the defective component must be transported to the third maintenance level — Depot — where more thorough repair is conducted. The organizational and intermediate maintenance is conducted at air stations or aboard aircraft carriers. Depot maintenance is performed at industrial air stations, at avionic repair facilities, and at civilian contractor sites.

All maintenance levels are supported by an associated supply activity. For the purposes of this thesis, the organizational level is considered to be an operational squadron at a major air station. The supporting supply department is the authorized interfacing mechanism for the squadron, the IMA, the depot, and the supply system.

2. Indenture Levels

An inspection of a weapons system reveals a hierarchy relationship among parts. This relationship is termed an Indenture Level because each of the major components within a weapons system has its own inventory of sub-assemblies. These levels are defined as follows:

Weapon Replaceable Assembly (WRA) - spare parts that are removed and replaced at the Organizational Level.

Shop Replaceable Assembly (SRA) - a sub-assembly of an WRA, repaired at the IMA and Depot Levels.

Consumable Repair Parts (Piece Parts) are used to repair the SRA's.

In order to visualize the relationship, one may consider the removal of a WRA as causing a hole in the aircraft. The absence of the WRA may not necessarily make the aircraft unsafe to fly, but it may result in the airplane being considered not operationally "ready" to perform its complete mission.

The supporting supply activity is authorized to maintain an inventory of WRA's, called a rotatable pool, which is based upon the expected WRA failure rate and the predicted repair turn-around time (RTAT) of the IMA. No WRA is authorized for stockage if the IMA has no repair capability for that particular WRA.

If a WRA is available in the rotatable pool, the squadron fills the WRA hole, and the aircraft maintains its readiness. Naturally, if a replacement is not available, the WRA hole remains until the IMA can repair and return the failed WRA to the squadron via the supply activity. The IMA repairs the WRA by analyzing the appropriate SRA's and by using applicable piece parts to repair the SRA's. The supply activity may also have a rotatable pool of SRA's to support the IMA.

The time that the aircraft remains unable to perform its mission is of vital importance to the operational commander. The time will normally be a function of the WRA rotatable pool stock level and the RTAT, but it can be extended considerably if the defective WRA cannot be repaired by the IMA. Then the failed unit is transported to the activity performing depot-level maintenance, and a replacement WRA is requisitioned from the supply system. Figure 1.1 depicts the material flow and the cycle times described above.

C. THE PROVISIONING CYCLE

1. The Preparation Stage

Most critical for a successful provisioning is the quality and the amount of advance planning that is performed. From the time that the Chief of Naval Operations establishes a new operational requirement, it is necessary that the logistic planners consider each developmental phase in light of its impact on the Navy support capability at the time of

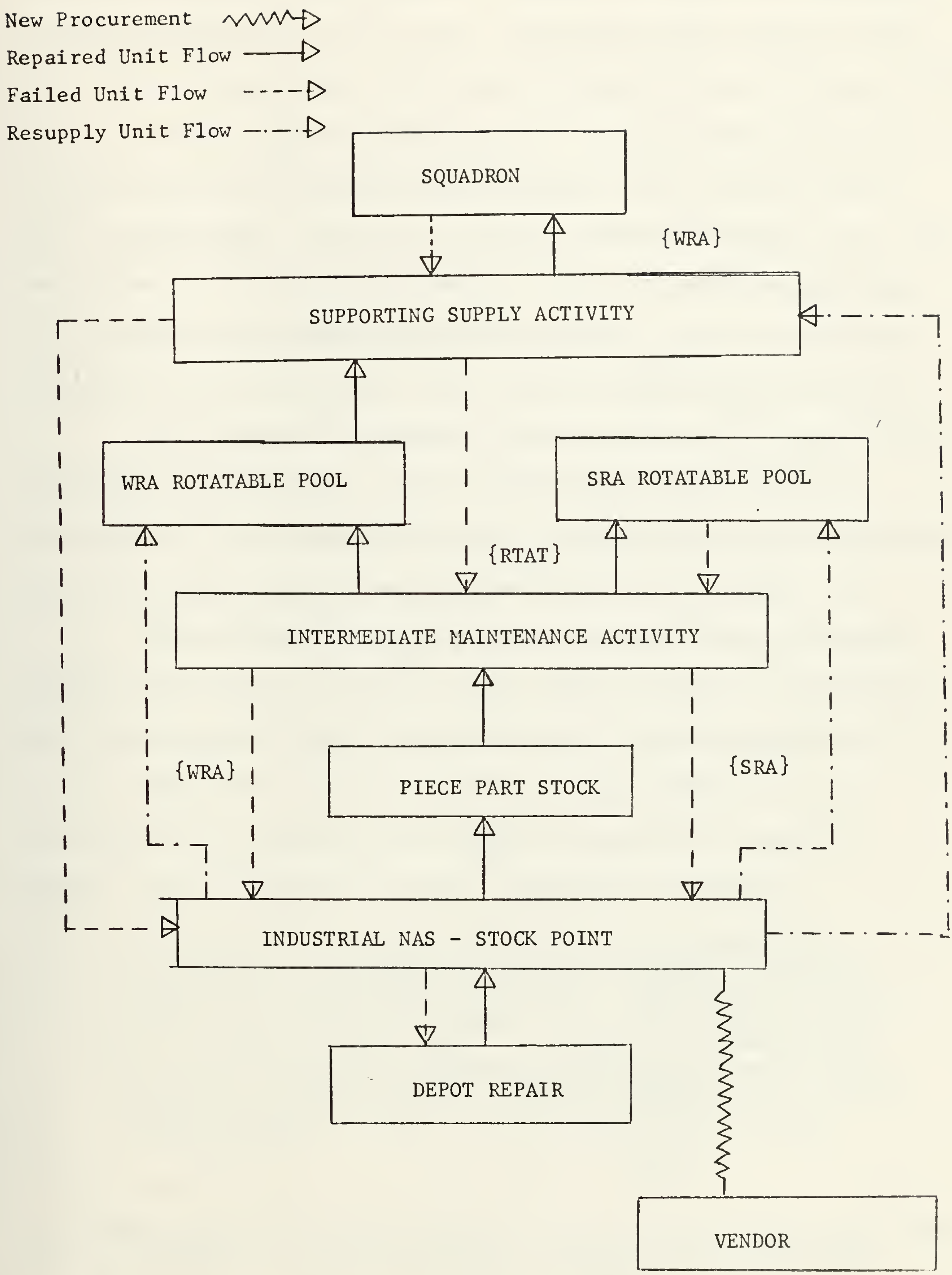

FIGURE 1.1 Material Flow and Cycle Times

Material Support Date (MSD).[1] Failure to give an inventory manager sufficient lead time — eighteen months prior to MSD — is cited as the major complicating factor in provisioning.[2]

After CNO notification, the Chief of Naval Material (CNM) assigns an Acquisitions Manager (AM), who then looks to the Commander, Naval Air Systems Command (NAVAIR) for the assignment of a Logistics Manager (LM). The LM establishes a Logistic Requirements Generation Team, and it is on this team that the coordination for provisioning actually begins.

ASO assigns a Weapons Manager (WM) to the team. His initial responsibility is to prepare the Supply Support Management Plan (SSMP) which is, essentially, a comprehensive list of supply support milestones leading up to the MSD.

The WM will insure that all Requests For Proposals (RFP) forwarded to civilian contractors contain requirements for provisioning data and provisioning conferences (Ref. 1).

The RFP will also include a requirement for the contractor who receives the award to conduct a Level of Repair (LOR) analysis program. Reference 2 states,

> LOR analysis is a justification of the decision to repair or discard the failed assembly for each anticipated maintenance action. This justification shall be provided to support the decision to repair at any maintenance level. Economic considerations are required except where over-riding, non-economic criteria can be cited... The LOR decision can affect manning levels, support equipment, stock levels, and training.

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[1]Navy Material Support Date is the date on which the Navy System Command/Inventory Manager assumes entire logistic responsibility for the new weapons system.

[2]Telephone conversation of 4 Aug. 1975 with Mr. T. Ianna, ASO, code WLL2.2.

The contractor is expected to begin the LOR analysis as soon as the preliminary design is determined and to continue revising the analysis through to final hardware design approval. Decisions resulting from the LOR analysis will strongly influence the maintenance plan, weapons system effectiveness, and total life cycle cost of the system.

Another facet of the RFP is the requirement for the contractor to provide augmented support prior to the provisioning conference. This allows for the design to stabilize and for the contractor to collect data necessary for use at the provisioning conference.

The WM will schedule the provisioning conference with sufficient notice to allow the contractor to submit the weapons system parts list to the Defense Logistic Support Center (DLSC) for screening. The screening will ascertain if any of the spare parts have been assigned a Federal Stock Number (FSN).

2. <u>The Provisioning Conference</u>

The provisioning team will meet at the contractor's design facility for the conference. The team chairman is the WM who will compose the team of representatives from:

NAVAIR
NAVAIR Representatives, Atlantic and Pacific
ASO Technical Division
Fleet Commands
Naval Air Rework Facilities
Contractor

As the first item on the agenda, the team will devote its attention to item coding. A Source, Maintenance, and Recoverability (S,M,&R) code is assigned to each item indicating the method of procurement, the lowest maintenance level authorized to remove and replace the item, the lowest maintenance level authorized to repair the item,[3] and the method of recoverability. Then based on the results of the DLSC screen, the provisioning team will request that all new items to the supply system be assigned a FSN. To assist in the assignment, the team will recommend the Federal Supply Class (FSC) to which the spare part should be assigned. This will then determine the inventory manager. It is ASO policy to maximize the integrated coding of consumable repair parts to the Defense Supply Agency.[4]

Once items are coded, the provisioning team turns to the failure prediction phase, its most important function. Equipment reliability estimates are furnished by the contractor as results from testing during the augmented support period. The estimates are stated in terms of:

Mean Time Between Maintenance Actions (MTBMA) -

> The time between maintenance actions, either preventive or corrective, at the IMA level.

Mean Time Between Failures (MTBF) -

> equipment operating time before depot repair is required.

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[3]This code assignment is based upon the LOR recommendation from the contractor.

[4]Reference 1.

Reference 3 analyzes current and possible alternative methods for designating failure rates for provisioning. The A7E provisioning, according to Ref. 3, resulted in an insufficient supply support because the failure predictions conformed to the reliability design targets which were much higher than what was actually experienced initially. See Figure 1.2.

Reference 4, however, reports that, because of pessimistic failure rates used in the provisioning of the F111, approximately 9.6 million dollars of excess spare parts were procured. This U.S. Air Force study found higher correlation between contractor failure predictions and the operational results than the correlation between the provisioner's estimates and the operational results.

Even though some attempts have been made to standardize the provisioning methodology, it appears that each provisioning conference is unique to the system under development and the participants.[5] The provisioning team must decide to accept or to degrade the contractor's failure predictions from which the team computes Maintenance Replacement Factors (MRF) and Rotatable Pool Factors (RPF) that are integral parts of the inventory selection procedure. As design concepts are standardized and the trend towards commonality continues, better utilization of the Mean Family Replacement Factor (MFRF) file, assembled from 3M data, can lead to higher validity in failure predictions.

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[5] Reference 3.

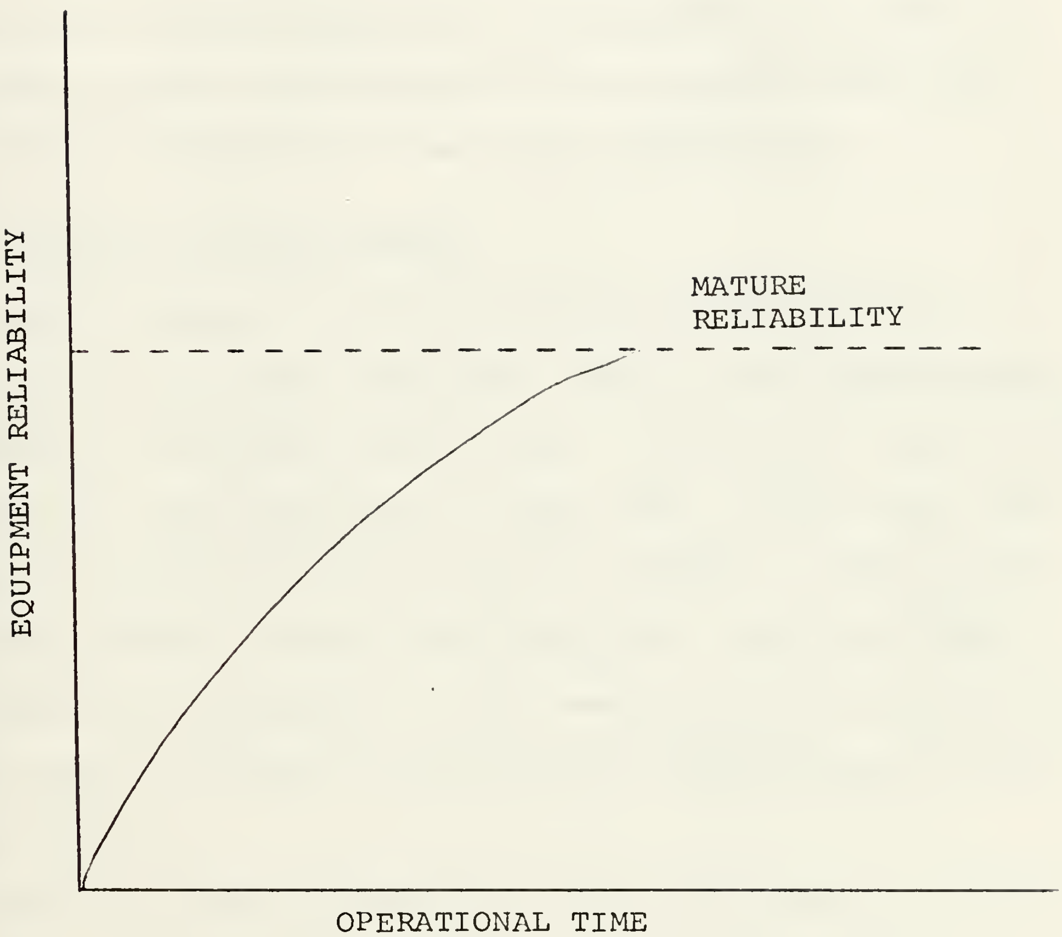

FIGURE 1.2 Reliability Results During Initial Support Period

For the purpose of provisioning, even though equipment design has not stabilized, the current design is considered "frozen." Future changes in design and possible parts applications are controlled through the Design Change Notice system.[6] The ASO WM is also assigned to the NAVAIR Aircraft Configuration Control Board for this purpose.

The results of the provisioning conference are recorded by the contractor who is responsible for updating the item data and for forwarding the records to ASO where the inventory selection is made (see chapter II).

D. MEASURES OF EFFECTIVENESS

Several measures of effectiveness are used in Naval Aviation, but the most meaningful measure to an operational commander is aircraft availability. Aircraft are unavailable if they are not operationally ready (NOR), which can be the result of pending maintenance action (NORM) or spare parts shortages (NORS). Since both conditions — NORM and NORS — may exist simultaneously for the same aircraft, availability is not explicit enough to evaluate the effectiveness of the supply support.

Parts shortages, however, do relate directly to unfilled demands on the supply department's repair part inventory. For the purpose of optimizing the operational site's inventory, shortages are considered to be backorders on the system. As it will be shown later, the shortage could result from an

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[6]Reference 1.

outstanding supply requisition or from a pending repair action. There is a unit shortage, and its replacement is either in the supply or repair pipeline. Whatever the source of the replacement, the shortage can be termed to be in a backorder status.

The measure of effectiveness (MOE) most extensively used in provisioning is the Awaiting Parts (AWP) rate. A unit or sub-assembly whose repair is delayed because of a parts shortage is back-ordered or in an AWP status. The AWP rate is the ratio of the number of units backordered to the number of major units supported. The cumulative effect of backorders at all indenture levels can be related to equivalent backorders of the parent WRA. This concept enables the inventory selection optimization technique, discussed in Chapter II, to maintain its credibility as an effective method.

Supply effectiveness is typically measured by FILL rate, which is the ratio of repair parts demands filled immediately from stock on hand to the total number of demands. Reference 6 studied the relationship of FILL rates to NORS rates at the base level and found very little correlation between the two. The authors recommended that results should be stratified by maintenance squadrons and shops, rather than composite base FILL rates, and that analysis of bench stock demands should be compared with NORS rates thirty days later. The FILL rate computation of Chapter II does stratify the demands as those pertaining solely to the weapons system being provisioned. Consequently it should be a meaningful MOE.

E. INITIAL OUTFITTING LISTS (IOL)

The inventory selection process is perpetrated at ASO and is published in the final form as an allowance list for the operating site(s). This inventory allowance publication is entitled an "Initial Outfitting List (IOL)." As can be expected from an allowance list, the IOL will list the item nomenclature, FSN, unit cost, SM&R code, and the number of applications. In addition, the spare parts inventory allowance is expressed as being directly related to the aircraft utilization time, expressed in flight hours per period.[7] An example is shown in Figure 1.3. Columnar headings for utilization rates may vary from one to eleven.

The IOL is divided into specific parts which concern the maintenance capability at each operating site. There are fifteen different parts which can be included in an IOL, and these parts can be segregated in ten different ways. For instance, Part I of the IOL recommends a range and depth of spare parts that cannot be repaired at the IMA,[8] while Part II lists those parts that are needed for IMA repair.[9] The unique feature involved is the concept of failures being directly related to operation utilization rates.

Chapter I reviewed the Navy maintenance and supply support system, the stages of the provisioning cycle, as well as

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[7]Usually 90 days.

[8]Compiled from MRF's computed at the provisioning conference.

[9]Compiled from RPF's.

Item No.	Accountability	Federal Stock No.	Nomenclature	U/I	Occurrence	Operating Hours 1-1200	2001-3000	3001-6000	Normal Usage	Effectivity Flag
532	R	2RH1430-152-2539AY SMR PIGGO	CKT CD ASSY 97942-617R56 $250.00	EA	1	0	1	1		

FIGURE 1.3 Example of IOL Publication

various measures of effectiveness that are used to select inventories for an initial outfitting list. Chapter II will discuss the IOL optimization model in use at ASO.

II. ASO IOL OPTIMIZATION MODEL

This chapter will review that portion of the ASO Provisioning Model that determines the range and depth of spare parts that are positioned at an operational site. The model was developed by General Dynamics and was tested along with other candidates by a joint committee composed of representatives from the Naval Supply Systems Command, NAVAIR, ASO, the Fleet Material Support Office, and the Weapons System Analysis Office.

The first section considers the mathematical basis for the model. The conceptual steps in the IOL computation are treated in the second section, and the last section comments on the options that are available to management in exercising the model.

A. MATHEMATICAL BASIS FOR THE ASO IOL MODEL

The ASO IOL Model is a mathematical representation of the Naval Aviation maintenance and supply support systems. The first section will briefly discuss the fundamental assumptions inherent in the model, and the second section will describe the mathematical relationships that are tools of the optimization scheme.

1. Assumptions

A viable inventory selection model should assume that the demand for spare parts is stochastic. In fact, large over-buys in provisioning can occur if a deterministic model

is used [Reference 5]. The probability distribution that seems to describe best low volume demand is the Poisson distribution [Reference 7]. Implicit in the choice of this distribution is the assumption that the variance of the demands is equal to the mean demand. This may be an adequate representation for low volume demand at an operational site, but it may be optimistic when the variance-to-mean ratio is different from unity. When the demand is aggregated at the Inventory Control Point (ICP) level, other probability distributions may be more appropriate. More is said about this in Chapter IV. The mean demand rate over the IOL Support period is assumed to be constant.

The times for resupply from the depot level and replenishment from the supply system are assumed to be independent of the influence of operational demand. The real world and the model can make adjustments for accelerated transportation to the operational unit, but the resupply time is more a function of material availability than the mode of transportation.

One-to-one requisitioning is assumed in the model. This should be reasonable because a squadron will, at a given time, normally have only one defective WRA which will either be inducted by the IMA or a replacement will be requisitioned. The requisitioning of piece parts may certainly not conform to this assumption because the relatively low-priced, high-volume, multi-application consumable spares would normally be resupplied at fixed intervals in larger quantities.

All demands are assumed to be satisfied eventually. Shortages are backordered.

Cannibalization is a term describing the maintenance practice of recovering serviceable spares from a defective unit that has a different parts shortage. These spares are then used to repair other assemblies and, therefore, to reduce the number of components in AWP status. Cannibalization is discouraged in Naval Aviation because the manhours invested in parts removal could be applied to other open maintenance tasks, especially since there is a finite probability that the parts shortage will be filled in the near future. The ASO IOL Model does not consider cannibalization as a maintenance alternative.

The IOL Model assumptions have been discussed and can be summarized as follows:

a. Poisson probability distribution of demand
b. Constant demand rate during the IOL period
c. Resupply time independent of demand
d. One-for-one requisitioning
e. Unsatisfied demands are backordered
f. No cannibalization

2. Mathematical Relationships

The mathematical model of a repair/resupply system utilizes the relationship between the number of spares, K, in the repair/resupply pipeline; the IOL stock, S; the expected number of backorders, B(S), for stock level S at the operational site; the IOL FILL rate, F(S); the AWP rate; and the equivalent WRA pipeline quantity, M_{WD}.

a. Expected Number of Spares in the Repair/Resupply Pipeline

Let K be the number of spares in the repair/resupply pipeline. Because of the original assumptions of one-for-one requisitioning and Poission distributed demands, K is Poisson distriubted with mean M [Reference 8].

IOL columnar spreads are expressed in terms of operating hours per site. Since the number of aircraft supported per site could vary, the common link in the utilization rate expressed in the IOL columns is the maintenance cycle. One maintenance cycle is equivalent to 100 operating hours per period.

The predicted failure rate provided by the contractor and either accepted or modified by the provisioners is used to compute the MRF and the RPF.

Other definitions and abbreviations that are applicable are:

M - Expected number of spares in pipeline that are not delayed for parts.

GRR - Gross Removal Rate

MTBF - Mean Time Between Failures (for depot repair)

MTBMA - Mean Time Between Maintenance Action (IMA)

MFBR - Mean Flight Hours Before Removal

FHF - Flight Hour Factor = $\frac{\text{ground operating time}}{\text{actual flight time}}$

FUR - Flight Utilization Factor = $\frac{\text{actual on time}}{\text{actual flight time}}$

MRF - Maintenance Replacement Factor; mean demand during a maintenance cycle for attrition items

RPF - Rotatable Pool Factor; mean demand during maintenance cycle for IMA repair processing

RTAT - Repair Turn-Around-Time; mean IMA repair time with no parts shortages

RST - Resupply time

MC - Number of Maintenance Cycles per period.

At the provisioning conference, the contractor furnishes estimates of the MTBMA's at the IMA and the MTBF's. The expected number of spares in the repair/resupply pipeline is computed as follows:

First, the mean flight hours before removal is calculated,

$$MFBR = \frac{MTBMA \cdot FHF}{FUR} .$$

Then the gross removal rate is computed as

$$GRR = 100 \text{ hr./maint cycle} \cdot \frac{1}{MFBR}$$

Once removed, the replacement will depend on work performed at the IMA or within the supply system. In order, therefore, to consider the number of spares in the pipeline, the IMA repair portion — RPF — of the removals must be computed, as well as the attrition portion — MRF.

$$MRF = 100 \text{ hr./maint cycle} \cdot \frac{FUR}{MTBF \cdot FHF}$$

$$RPF = GRR - MRF .$$

Obviously, the rotatable pool factor and the maintenance replacement factor are not independent; they are the main ingredients that comprise the average number of spares in the pipeline with no parts shortage. The appropriate relationship is:

$$(2.1) \qquad M = \frac{MC}{90} \quad \{\underbrace{\overset{\text{IMF}}{RPF \times RTAT} + \overset{\text{SUPPLY}}{MRF \times RST}}_{\text{PIPELINE}}\} \;.$$

The probability that there will be K units in the pipeline is:

$$P(K) = \frac{M^k e^{-M}}{K!} \qquad K = 0,1,2,3,\ldots$$

b. Expected Number of Backorders

Let K equal the actual number of spares currently in the pipeline. As a reminder, that means that K units are <u>still</u> in repair at the IMA or have been requisitioned from the supply system. Units that have been returned for depot repair lose their organizational source identity and once repaired are returned to the supply system, from which a requisition can be satisfied.

Let S equal the IOL stock level positioned at the operating site. If K is greater or equal to S, there are no more replacements to satisfy organizational demands. Future demands will result in backorders. Then the expected number of backorders is:

$$(2.2) \qquad B(S) = \sum_{K=S+1}^{\infty} (K-S)\, P(K) \quad .$$

One may question the upper limit of the summation. Since there are N units (aircraft) supported, K would not normally exceed N+S units. It would be highly improbable since N, the number of aircraft supported, normally is much larger than the mean demand M. The probability of K approaching N+S is very low, and consequently little error is introduced by using expression (2.2).

Obviously, if the IOL stock level S were zero, then

$$B(0) = \sum_{K=1}^{\infty} K\, P(K) \;=\; M \quad .$$

c. IOL FILL Rate

The IOL FILL rate can be expressed as the percentage of demands that can be satisfied from the IOL stock levels. This is equivalent to the probability that a demand will be satisfied with no delays. This event will occur if and only if the number of units in the pipeline, K, is less than or equal to S-1. Therefore, the IOL FILL rate is given by:

$$(2.3) \qquad F(0) = 0$$

$$F(S) = \sum_{K=0}^{S-1} P(K) \;=\; \sum_{K=0}^{S-2} P(K) + P(S-1)$$

$$= F(S-1) + P(S-1)$$

$$F(S+1) = F(S) + P(S) \quad . \tag{2.3a}$$

d. The Recursive Relationship for Backorder Computations

Recall that the expected number of backorders, equation (2.2), is

$$B(S) = \sum_{K=S+1}^{\infty} (K-S)\ P(K) = 1 \cdot P(S+1) + 2 \cdot P(S+2) + \ldots \quad .$$

Then

$$B(S+1) = P(S+2) + 2 \cdot P(S+3) + 3 \cdot P(S+4) + \ldots \quad ,$$

and by subtraction we have

$$B(S) - B(S+1) = P(S+1) + P(S+2) + \ldots = \sum_{K=S+1}^{\infty} P(K) \quad .$$

From equations (2.3a), the IOL FILL rate is

$$F(S+1) = F(S) + P(S) \quad ,$$

and recalling that

$$\sum_{K=0}^{\infty} P(K) = 1 \quad ,$$

then

$$B(S) - B(S+1) = \sum_{K=0}^{\infty} P(K) - \sum_{K=0}^{S} P(K) = 1 - F(S+1)$$

$$= 1 - \{F(S) + P(S)\} \quad .$$

The following recursive relationship is used to compute the expected number of backorders with one more item of stock added to the IOL:

$$\text{(2.4)} \qquad B(S+1) = B(S) - 1 + F(S) + P(S)$$

e. The Relationship Between Indenture Levels and Backorders

Equation (2.1) expressed the mean number of spare parts in the repair/resupply pipeline as a function of the sum of the rotatable pool demands and the attrition demands. This expected number, M, includes only those WRA's and SRA's that have no subordinate parts shortages. The effect of a parts shortage is twofold:

1. a backorder for the lower indenture item, and
2. an increase in the number of higher indenture items in the pipeline.

Assuming only one backorder for each SRA that is delayed, then the total number of spares in the pipeline, with and without delays, is

$$(2.5) \qquad M_{WD} = \frac{MC}{90}\left[RPF \cdot RTAT + MRF \cdot RST\right] + \sum_{i=1}^{N_s} B(S_i) ,$$

$$= M + \sum_{i=1}^{N_s} B(S_i)$$

where N_s = number of lower indenture items

and $B(S_i)$ = expected number of backorders for the i^{th} lower assembly with stock level S_i.

Precisely the same relationship exists for the WRA's.

f. AWP Rate

A WRA backorder implies that an end item is in AWP status, i.e., a hole still exists in the aircraft. The AWP rate at the squadron level can be approximated by the ratio of WRA backorders to the number of end items supported. Thus

$$\text{AWP Rate} = \frac{\sum_{j=1}^{N} B(S_j)}{EI} ,$$

where $B(S_j)$ = expected number of backorders for the j^{th} WRA,

N = number of WRA's in the IOL,

and EI = number of end items supported.

This completes the description of the mathematical basis for the ASO IOL model. Section B will discuss the conceptual steps of the IOL computation.

B. CONCEPTUAL STEPS IN THE IOL COMPUTATION

This section will present the various conceptual steps used in computing the optimum IOL inventory level that produce the desired supply effectiveness for the least investment. An examination of the objective function will also entail a discussion of the backorder penalty costs, followed by a description of the inventory selection procedure. The last part will review the procedures the model uses to treat range and depth constraints, common items, and rotatable pool allocations.

1. Objective Function

The optimization technique utilizes the relationship, discussed in the previous sections, of parts shortages of lower indenture items to WRA backorders. The optimum IOL stock level is taken to be that which causes each WRA and its inventory of sub-assemblies to make the same cost-effective contribution to the IOL inventory. The idea, then, is to minimize the expected number of WRA backorders per dollar invested. Therefore, the backorder penalty cost has to be compared with the cost of having the inventory in the first place. Even though the model does not use a holding cost factor for inventory, it is implicit in the concept because the objective function may also be expressed as:

$$\text{MIN} \quad C_o(\overline{x}) = \sum_{i=1}^{NW} \{\lambda \cdot WB(\overline{x}_i) + IC(\overline{x}_i)\} \quad ,$$

where

$\overline{x}$ = IOL inventory vector,

$C_o(\overline{x})$ = cost of inventory $\overline{x}$,

$\overline{x}_i$ = Inventory of i^{th} WRA and sub-assemblies,

$IC(\overline{x}_i)$ = Investment cost of inventory $\overline{x}_i$,

$\lambda \cdot WB(\overline{x}_i)$ = Backorder penalty cost for inventory $\overline{x}_i$,

and NW = number of WRA candidates.

Figure 2.1 depicts the trade-off relationships that are necessary to achieve the IOL optimization.

2. Backorder Penalty Cost Multipliers

The backorder penalty cost, λ, is a Lagrangian multiplier which unifies the inventory selection criteria over all WRA's and their lower indenture parts. The multiplier, λ, has the dimensions of cost per backorder. The particular value of λ is initially selected by management (discussed in the next section). Figure 2.2 is an example of the information used in this selection process.

It is recognized that the optimum inventory (desired AWP target for least investment) may result in a slightly different inventory cost or expected AWP rate because of the discrete values that are used as AWP rate targets or inventory cost targest (see Section II.C). The λ associated with the target may not be precisely the one that corresponds to the discrete value selected. The IOL computer program will recognize this difference and recompute by interpolation a new backorder penalty cost multiplier. A new IOL inventory

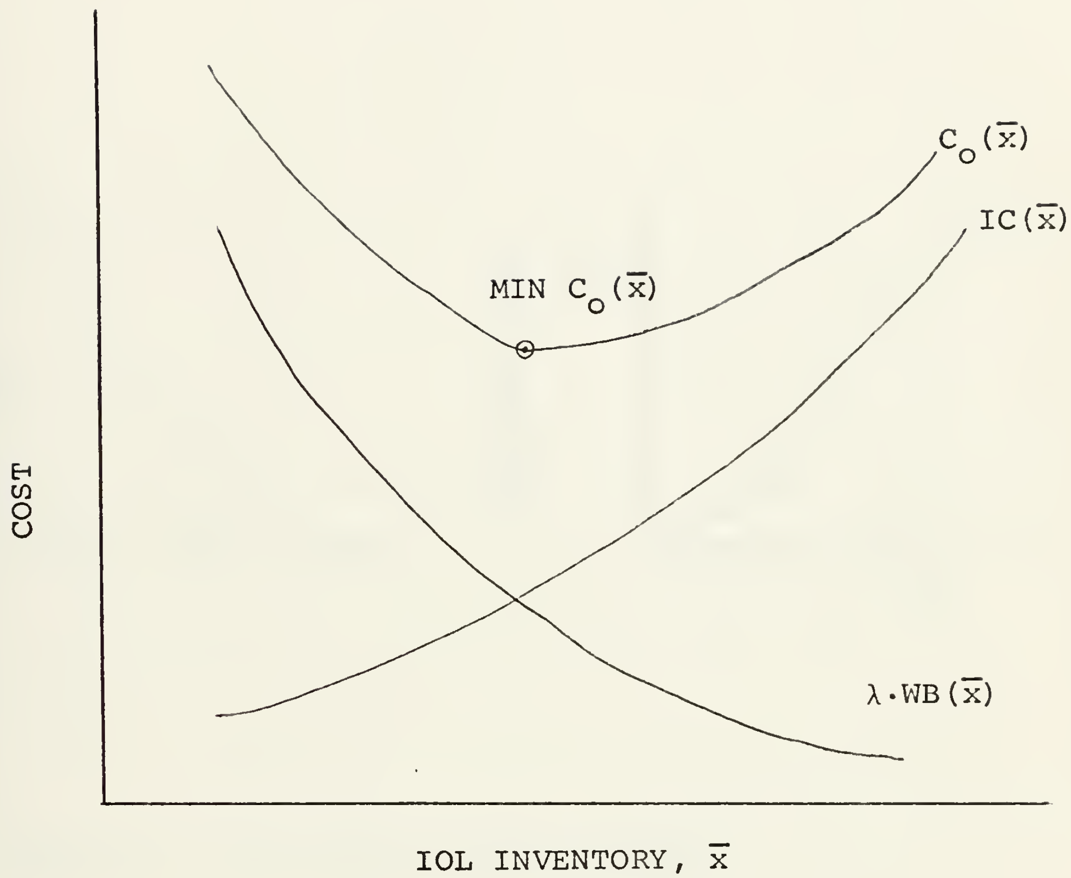

FIGURE 2.1 Trade-Off Relationship in IOL Optimization

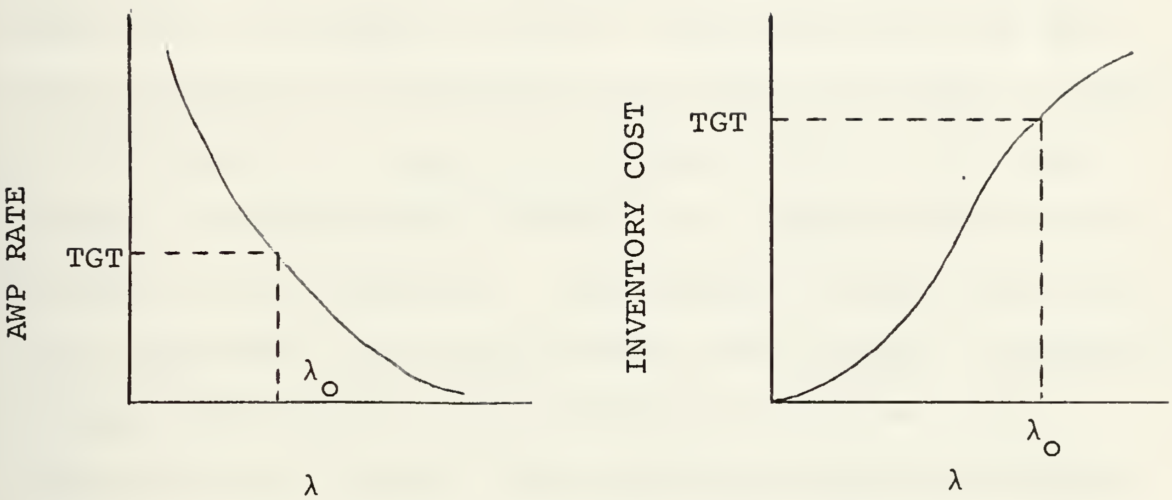

FIGURE 2.2 Selection of Multipliers

is selected. If the resultant AWP rate or inventory cost is still different from management's target, it is assumed that the desired solution does not exist and the last solution is accepted.

3. Inventory Selection Procedure

Marginal analysis[1] is used to select items for the IOL inventory. It is assumed that the addition of one item at any indenture level will cause a decrease in WRA back-orders (See paragraph II.A.2d). The spare part with the largest decrease in backorders per dollar invested is added to the inventory. Cost-effectiveness ratios (unit cost)/(change in WRA backorders (ΔEBO)) are computed after each addition to the inventory. The marginal analysis continues until the lowest cost-effectiveness ratio equals or exceeds the backorder-penalty-cost multiplier, at which time computations are ceased. The next WRA is then considered and the process is repeated.

Since the objective function is stated in terms of minimization of either costs or WRA backorders and since both are convex under the assumptions of the model, a marginal analysis technique will derive a solution to the inventory problem which is nearly optimal. The addition of one more item of stock will certainly increase the holding cost, but until the minimum point of the objective function is reached, the number of expected WRA backorders will decrease at a faster rate and, thusly, make it worthwhile to stock the item.

---

[1]Reference 13.

A complete step-by-step procedure follows:

a. Calculate the pipeline quantity, M (Equation 2.1), for for every part (WRA, SRA, and PP).

b. Set the expected backorders, B(0), for a zero stock level equal to M.

c. Calculate M_{WD} (Equation 2.5) for every item that has a lower indenture item.

d. Temporarily add one part to the inventory.

e. Determine the change in WRA backorders.

f. Compute the cost-effectiveness ratio,

$$C/E = \frac{\text{item unit price}}{\text{EBO}} \text{ ,}$$

and store the value.

g. Continue steps d, e, f for every other part in the WRA group. Rank the cost-effectiveness ratios.

h. Select the spare part with the lowest ratio if it is less than λ, the backorder penalty cost multiplier.

i. Update M_{WD} and EBO for all higher indenture parts of the spare part that was added.

j. Repeat steps d through h until the C/E ratio equals or exceeds λ.

An example of this procedure is illustrated in Appendix A.

4. Range and Depth Constraints

Even if there were no budgetary constraint on the IOL optimization, there are very practical reasons to consider restricting the range and depth of the inventory. Estimates of the number of line items that are candidates

for stockage to support an operating site are in the neighborhood of 200,000. It is not feasible from both a space and a manpower consideration to accept such a large inventory. Some logical decision rule must be applied to restrict the range if the normal budgetary constraint does not interfere first.

Within the imposed financial boundary, the IOL optimization technique would tend to buy more low-cost items. Beyond a certain point, greater depth may not be meaningful. The IOL program restricts depth rather easily, as will be shown.

a. Range Restriction

The Aviation Supply Office restricts the range of stock candidates by ensuring that each item stocked will have a mean expected demand greater than a given minimum amount. This demand "floor" is a function of unit cost and application. For instance, for avionics applications involving items costing less than $5,000, the demand floor is one demand in nine months. Parts costing more than $5,000 require a demand frequency of once in six months. Ground support equipment (GSE), however, is stocked if it has an expected volume of one in sixteen quarters. The demand floor criteria for the non-GSE items were set arbitrarily to conform to the expected length of an aircraft carrier's cruise deployment.

The possibility exists that it may be less than optimal to restrict items on this basis. The IOL optimization

model, however, utilizes the same type of marginal analysis to reduce the inventory range as was used in the inventory selection phase. The program proceeds as follows:

1. The number of line items, N, that would have been eliminated under the procedure described above is determined.

2. The full inventory range is selected, as outlined in Section II.B.3.

3. The spare parts which have no lower indenture level are considered as candidates for elimination from the inventory.

4. The affect of reducing the spare-part inventory to zero is to increase the number of WRA backorders. The program then computes the change in expected backorders.

5. The net increase in backorders is computed as follows:

$$\text{Net Increase in WRA Backorders} = \Delta\text{EBO} - \frac{\text{number of items eliminated} \cdot \text{unit cost}}{\lambda}$$

As previously noted, λ is the backorder-penalty-cost multiplier. This net increase suppresses the influence of the rise in WRA backorders by taking into account the funds made available for other inventory investment.

6. The spare parts are then ranked according to their impact on the WRA backorders.

7. The program then selects and eliminates N items — the number that would have been eliminated by the ASO demand floor rules — with the lowest net increase in WRA backorders.

Examples illustrated in Reference 7 indicate that a smaller reduction in effectiveness (AWP rate) will occur with the IOL optimized range reduction technique than under the ASO decision rules formerly used.

b. Depth Restriction

It is conceivable that with sufficient procurement funds available the IOL optimization program could select an abundance of low-cost items and effectively reduce the expected backorder value to an insignificant level. The program will cease to consider an item as a candidate for further stockage when its EBO reaches an arbitrary small level, e.g.

Indenture Level	Backorder Constraint
WRA	.001
SRA	.002
PP	.003

5. Common Items

Spare parts that have multiple applications are defined as common items. Multiple applications within the same WRA are considered as one occurrence and demands are aggregated. For each common item, expected backorders over all occurrences, B, are summed as is the expected number of units, D, in the pipeline. A consolidated inventory S_c, is computed relative to D such that EBO is less than B. The inventory, S_c, is then allocated to each WRA with the allocation decisions based upon the ratio of applications in each WRA to the total number of applications in the weapons system.

Common items that have no lower sub-assemblies are treated as candidates for elimination in the same manner as discussed in Section II.B.4.a.

6. Allocation for Rotatable Pools

Reparables for which the IMA has repair capability will have both a rotatable pool and attrition (MRF) factors. Once an optimum inventory level, S_i, is determined, a percentage is allocated as rotatable pool inventory based upon the ratio of rotatable pool pipeline to the combined repair/resupply pipeline. The rotatable pool inventory level for the i^{th} item is:

$$S_{R_i} = S_i \left\{ \frac{RPF \cdot RTAT}{RPF \cdot RTAT + MRF \cdot RST} \right\} ,$$

where S_i is total inventory of i^{th} item,

and S_{R_i} is rotatable pool inventory for i^{th} item.

The balance of the inventory is attrition stock.

Section B has considered the IOL optimization model's objective functions, its use of backorder penalty cost multipliers, and its computational technique. Also discussed were range and depth restrictions, common items, and rotatable pool inventories. Section C will review briefly some of the options that management may specify in using the model.

C. MANAGEMENT OPTIONS

The ASO IOL Optimization model allows the ASO management to utilize either of two options in order to manage the inventory selection. These options allow the ASO to specify "targets" which indicate whether supply effectiveness or procurement costs are the over-riding concern.

1. AWP Target

The ASO IOL model computes an inventory level that will produce the same AWP rate for any utilization rate (all IOL columns). Management may specify up to 5 AWP targets, and the program output lists the cost by cognizance code for each AWP target.

2. Cost Targets

Three cost targets may be specified as constraints for the inventory selection, and up to three cognizance codes may be designated for each cost target. If only one cost target is specified and no cognizance code is listed, then that cost will be the total procurement ceiling for all cognizance codes.

Along with this option, a maximum acceptable AWP RATE is indicated. If the cost constraints produce inventory levels with expected AWP RATES that exceed the maximum acceptable AWP RATE, then inventory costs for each AWP RATE are printed for management's evaluation and decision.

This chapter discussed the Navy Maintenance and Supply support systems, the mathematical basis for the IOL model, as well as the conceptual steps necessary for the

inventory selection. "Real world" constraints or targets that ASO management must consider in the inventory selection process were also discussed.

The next chapter will show the results of a sensitivity analysis that was conducted utilizing the model and data supplied by NAVAIR.

III. SENSITIVITY ANALYSIS

Sensitivity analysis is a term that is commonly associated with operations research studies. In conducting the analysis, the goal is to identify the sensitivity of the optimal solution to the values of the various parameters. It may be that the optimal solution is very sensitive to values of one parameter, but insensitive to the magnitude of another parameter. By identifying the sensitive parameters, a decision maker can determine where he needs to concentrate his attention. In this chapter a sensitivity analysis of the model parameters that was conducted will be described.

A. TEST DESCRIPTION

Data concerning avionics sub-systems of the Harrier aircraft, provisioned by ASO, was used in the model to recommend inventory levels. The sub-systems contained 100 different line items, twenty-four of which were WRA's. All items had positive maintenance factors (RPF/MRF), and there were seventeen common line items. Parameters that were varied in the test were:

AWP rate
Resupply time
Repair-turn-around time
Range reduction floor rules

The test considered the influence of varying one parameter on the inventory investment cost in order to achieve the

comparable AWP rate of the initial setting. The initial values of the parameters were:

AWP rate	50%
Resupply rate	90 days
Repair turn-around-time	3 days
Range reduction floor rules	none

B. AWP RATE VS. INVENTORY LEVEL

Selecting nine AWP targets, the ASO IOL model recommended corresponding inventory levels for those targets. All other parameters were constant as mentioned above. Table 3.1 displays the minimum achievable AWP rates by AWP target, Columns one and two. Columns four and five compare the reduction of the AWP rate from the initial level (49%) with a marginal cost ratio, the inventory cost increase from the initial inventory divided by the base inventory ($1,466,700). For example, the AWP target of 25% recommends a $1.6 million inventory which predicts an AWP rate of 22%. The AWP rate reduction is 27 percentage points, but it calls for an investment increase of 10.3%. The added investment would probably be justifiable. Figure 3.1 depicts this comparison, and point A can be interpreted to be the point of zero marginal profit (approximately 5% AWP rate). Point A could be interpreted as the point beyond which no further investment should be made. Figure 3.2 shows another cost-effectiveness comparison, protection cost (total inventory investment/AWP protection[1]) versus predicted AWP rate. Point B is the point

[1]AWP protection = 1 - predicted AWP rate.

TABLE 3.1 AWP RATE VERSUS INVENTORY LEVELS

Target AWP Rate (1)	Minimum Achievable AWP Rate (2)	Inventory (3)	Reduction in base AWP Rate[1] (4)	MCR $\frac{\Delta \text{ cost}^2}{\text{base inv}^3}$ (5)	PROTECTION COST $\frac{\text{Total cost}}{\text{Total AWP Reduction}}$ (6)
50%	49%	$1,466,700	0	0	$28.759
40	43	1,479,600	6%	.9	25,958
30	29	1,558,200	20	6.2	21,947
25	22	1,617,450	27	10.3	20.761
20	22	1,621,200	27	10.5	
15	17	1,704,446	32	16.2	20,536
10	10	1,868,441	39	27.4	20.761
5	5	2,078,441	44	41.7	21,878
1	0	2,372,441	49	61.8	23,724

1. 49%, column 2
2. Change in cost from base inventory
3. $1,466,700

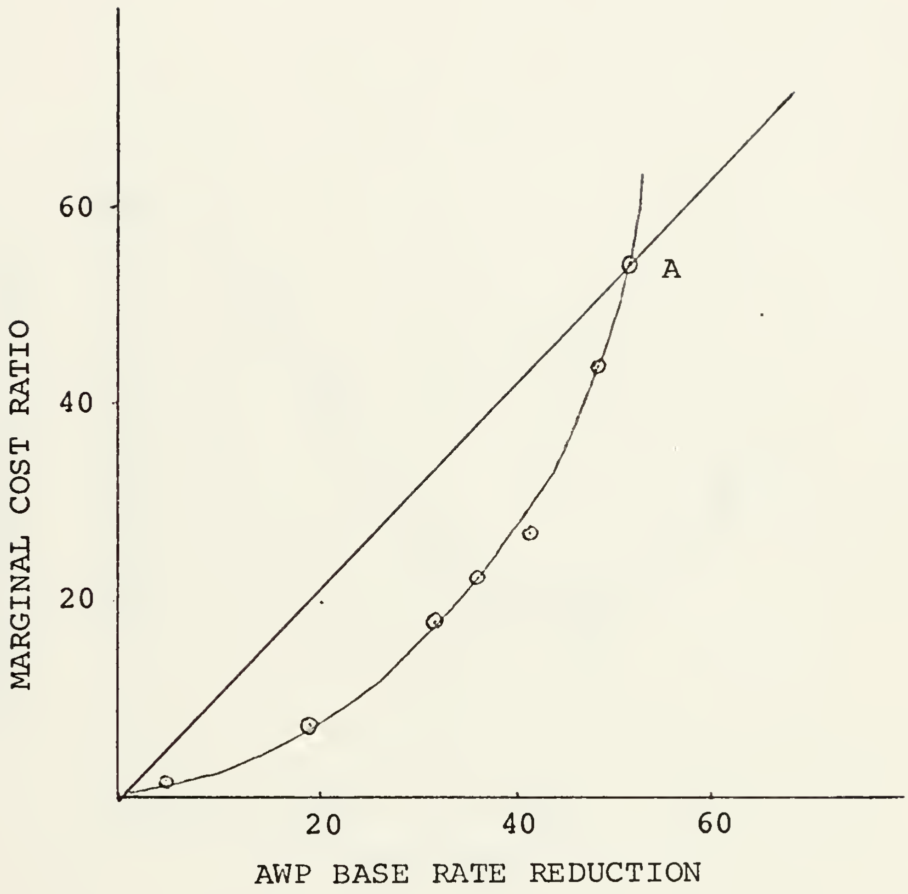

FIGURE 3.1 Marginal Cost Ratio vs. AWP Base Rate Reduction

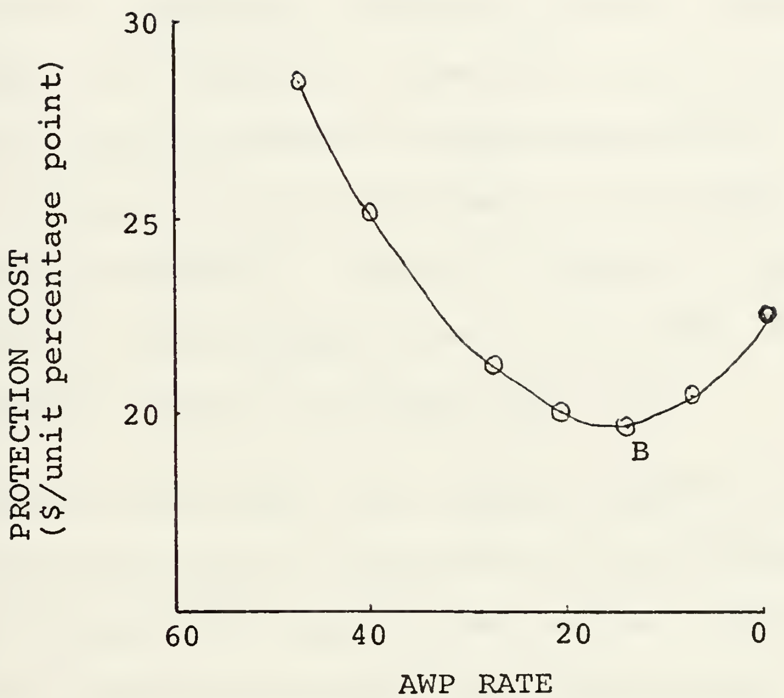

FIGURE 3.2 Protection Cost vs. AWP Rate

at which the minimum investment cost ($20,536/AWP %) is located (approximately 15-17%).

C. RESUPPLY TIME VERSUS INVENTORY LEVEL

The second parameter test investigated a range of the resupply time from zero days to ninety days. No stock range restrictions were used, and the repair time was fixed at three days. Table 3.2 lists the inventory levels for the various resupply times. The inventory levels were projected to produce a predicted AWP rate of zero. Column 3 of Table 3.2 shows the incremental inventory cost per day increase in resupply time to achieve the same protection. The range from ten to fifteen days RST resulted in the largest incremental inventory investment. Figure 3.3 depicts a linear growth in inventory as the resupply time is extended. Point C labels the region from ten to fifteen days RST.

D. REPAIR TURN-AROUND-TIME VERSUS INVENTORY LEVEL

The third test was concerned with the investment level required as the IMA repair level varied. The resupply time was held constant at ninety days and no range constraints were used. The test evaluated four values for RTAT and determined the inventory levels required to obtain predicted AWP rates of zero. Table 3.3 presents the results of the test; column (3) lists a cost-effectiveness ratio of incremental investment increase to the increase in repair time. Clearly, the largest investment to achieve similar protection was required when repair time is increased from

TABLE 3.2 RESUPPLY TIME VERSUS INVENTORY LEVELS

Resupply Time (1)	Inventory Level[2] (2)	CER Cost days (3)	% of Base Inventory[1] (4)
0	1,209,905	-	100
5	1,298,755	17,770	107
10	1,311,155	2,480	108
15	1,522,555	42,280	126
30	1,646,755	8,280	136
60	1,841,044	6,476	152
90	2,372,441	17,713	196

1. $1,209,905
2. Minimum achievable AWP - 0%

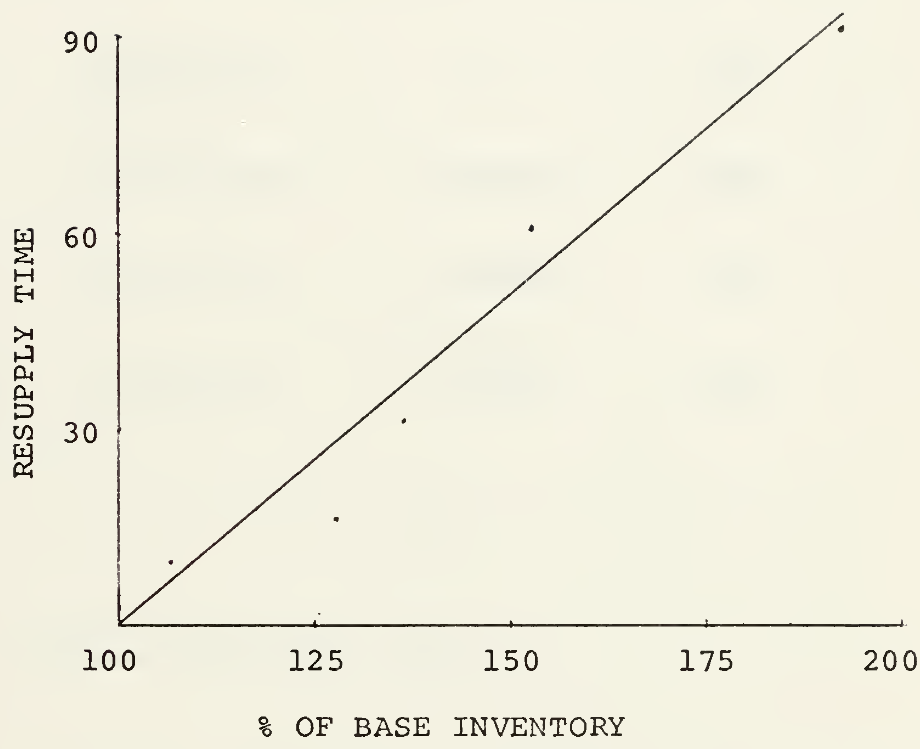

FIGURE 3.3 Resupply Time vs. % Base Inventory

TABLE 3.3 REPAIR TURN-AROUND-TIME VERSUS INVENTORY LEVELS

RTAT (1)	Inventory Level[1] (2)	CER $\frac{\text{cost}}{\text{days}}$ (3)	% of Base Inventory (4)
1	1,845,294	-	100
3	2,372,441	263,574	129
6	2,811,940	146,500	152
9	3,576,246	254,769	194

1. Minimum achievable AWP 0%.

one to three days. Figure 3.4 also shows a linear growth in inventory investment with the increase in repair time, but at a much faster rate than the growth due to extended resupply time.

E. RANGE REDUCTION VERSUS INVENTORY LEVEL

The final test analyzed the model's use of demand constraints that limit the range of line items in the inventory. Range restrictions are considered necessary by fleet commanders who may have storage limitations afloat. Holding resupply time and repair time constant at ninety and three days, respectively, three different criteria were used:

	CRITERIA				
Method	Demand		Unit Price		Demand
I	1 demand/9 months	<	\$5000	<	1 demand/6 months
II	1 demand/6 months	<	\$5000	<	1 demand/3 months
III	1 demand/3 months	<	\$5000	<	1 demand/3 months

Methods I and II resulted in slightly higher inventory levels and a 15% reduction in line items than Method III, which also had a 16% reduction. Table 3.4 compares the Method III results with the unconstrained test (90 days RST and 3 days RTAT). The range restriction technique tends to eliminate low-volume, low-cost items as inventory candidates. Consequently, for the same projected AWP target, inventory

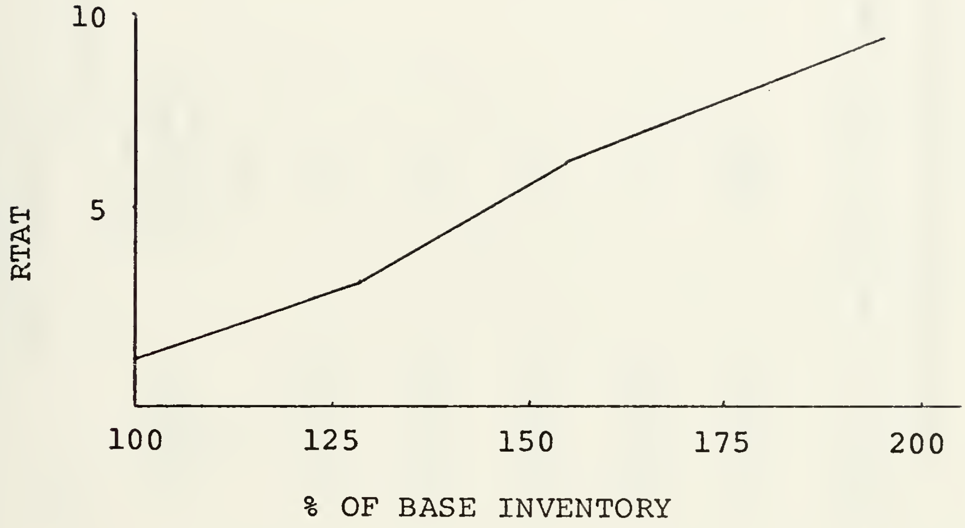

FIGURE 3.4 RTAT vs. % of Base Inventory

TABLE 3.4

	Range Reduction				
	Unconstrained		Constrained		CER
Target AWP Rate	Minimum Achievable AWP Rate	Inventory Level	Minimum Achievable AWP Rate	Inventory Level	$\frac{\text{Cost}}{\text{Base Inventory}}$
(1)	(2)	(3)	(4)	(5)	(6)
25	22	$1,617,450	25	$1,634,050	1.0
15	17	1,704,446	16	1,831,342	7.5
10	10	1,868,441	10	2,090,842	11.9
5	5	2,078,441	6	2,335,342	12.4
1	0	2,372,442	None	None	None

1. 16% line item reduction at each AWP Target Level.
2. Base inventory is the corresponding unconstrained inventory level, column 3.

costs are higher when range restraints are utilized. Column 6 of Table 3.4 displays a cost-effectiveness ratio as the incremental increase in cost (column 5 - column 3) to the base inventory (column 3). It is a management decision if a 12.4% investment increase is worth a 16% range reduction.

The purpose of these tests was to demonstrate that conducting sensitivity analysis of the parameters while using actual provisioning data could illustrate an optimum set of parameter values. A final test was performed using the following values:

AWP target	15%
Resupply time	10 days
Repair time	1 day
Range reduction floor rules	none

In order to achieve less than a 5% AWP rate for low utilization (first two IOL columns) and a 15% AWP rate for higher utilization, the resulting inventory recommendation was %580,355. This was a 66% reduction from the base inventory value shown in Table 3.1 for a 15% AWP rate. Of course, the question remains unanswered as to whether the $1.1 million dollars savings could be invested to reduce the resupply and repair times to the levels recommended.

This chapter described the results of sensitivity analyses applied to various parameters of the ASO IOL Model. Five analyses were performed using actual line item data for the S-3 aircraft and the results were summarized. The next chapter will investigate the use of an alternative probability distribution for demand to replace the Poisson distribution that is the basis for the IOL model computations.

IV. POISSON DISTRIBUTIONS

The ASO IOL model in Chapter II assumes that the number of demands, N(t), during any fixed time period, t, will be Poisson distributed with a mean demand, M. Since organizational units have no authorized backup stock, one-for-one requisitioning can be assumed at the organizational level, where WRA's only are removed and replaced.

Within the IMA, it is quite possible that one-for-one requisitioning may not occur at the SRA and piece-part level. Multiple applications within one defective WRA, as well as economic order quantities for piece parts, are reasonable examples of multiple quantities per demand. Actual demand data often shows variance/mean of greater than one. This is a reason to investigate alternatives.

This chapter will, by using actual ASO demand data, give examples of the discrepancies that can develop from incorrect distributional assumptions.

A. POISSON PROCESS

A Poisson process is one in which the number of events, N(t), occurring in a finite time period, t, satisfies the following conditions:

1. N(0) = 0.

2. The counting process during any fixed time increment is independent of previous time periods.

3. $N(t) \overset{d}{\sim} \text{Poisson}(\lambda t)$, which expresses the fact that the number of events occurring during a time period t at a rate of λ is Poisson distributed with a mean λt. For time periods s, $t \geq 0$

$$P\{N(t+s) - N(s) = k\} = \frac{e^{-\lambda t}(\lambda t)^k}{k!}, \quad k = 0,1,2,\ldots$$

It is easily shown that

$$E[N(t)] = \lambda t \quad \text{and}$$

$$VAR[N(t)] = \lambda t \ .$$

Examples of events that conform to a Poisson process could be[1]

arrivals of cars at an intersection,
customers entering a store,
radioactive disintegrations, and
chromosome interchanges in cells.

Reference 11 also shows that the interarrival times between events are a sequence of independent identically distributed exponential random variables with a mean of $1/\lambda$. From any point in time, this process, from a probabilistic sense, has no memory of previous occurrences.

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[1]Reference 9.

B. COMPOUND POISSON

The major difference in the compound Poisson process is that, while the inter-event time can be described as a Poisson process, the number of events occurring is random. Figure 4.1 shows a frequency distribution of the number of events occurring at random times.

A compound Poisson process is mathematically expressed by

$$X(t) = \sum_{i=1}^{N(t)} z_i \quad , \quad t \geq 0$$

$$N(t) \overset{d}{\sim} \text{Poisson}(\lambda t)$$

$\{z_i, i=1,2,\ldots\}$ is an independent, identically distributed family of random variables.

The mean of the compound Poisson process is expressed as

$$E[X(t)] = E[N(t)] \cdot E[Z]$$

$$= \lambda t \cdot E[Z] \quad .$$

The variance is given by

$$VAR[X(t)] = \lambda t \cdot E[Z^2] \quad .$$

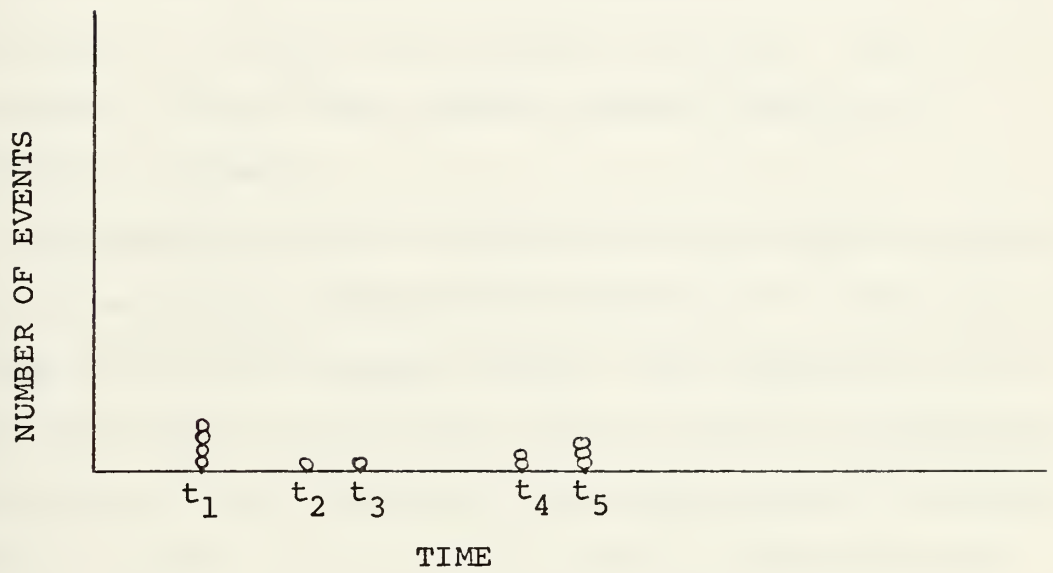

FIGURE 4.1 Pictorial Description of a Compound Poisson Process

A more convenient method to identify a single distribution from the variety of distributions that are members of the compound Poisson distribution is by the variance-to-mean ratio,

$$q = \frac{VAR\{X(t)\}}{E\{X(t)\}} \quad .$$

The compound Poisson is the generalized process where the variance is equal to or exceeds its mean. When q = 1, the specific case of the Poisson process described in Section IV.A arises.

"The compound Poisson distributions are the most general class of memoryless discrete distributions."[2] The implication is that at any time between events, the probability of an event occurring in the current time interval is independent of the time of the last event. This fact seems to conflict with the concept of preparing an IOL in accordance with maintenance cycles. An aircraft maintenance cycle, as may be remembered, is equal to one hundred flight hours. An increased flight-hour program assumes an increased number of equipment failures.

A study concerning costs of Naval Aviation conducted at NAS Oceana was completed in December, 1970, with the results approved, and then forwarded to CNO by Reference 12. Extracts are quoted for information:

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[2]Reference 10.

> The existing method of Flying Hour Program budgeting assumes that costs in all categories vary directly with each hour flown; conversely, logic and experience indicate that only fuel and lubricants and recording materials actually expended during flight bear any reliable direct relationship to the accumulation of flight hours... These results...indicate that the majority of maintenance costs per airframe are fixed regardless of the hours flown.

C. REAL DATA AND THE POISSON PROCESS

Data[3] relevant to thirty-one aviation items were received from ASO. Extracts as well as the computed mean, variance, and variance-to-mean ratio are shown in Appendix B.

Compound Poisson tables published in Reference 10 were used to compare the expected number of backorders for given inventory levels using the variance-to-mean ratio of units and of that actually computed for the item data. Since the tables were truncated, only items with a mean equal to or less than ten and a variance-to-mean ratio of seven or less were used. Figure 4.2 aggregates all item data. Figure 4.3 shows the EBO for all consumables; Figure 4.4, all reparables. Figure 4.5 depicts one item with a relatively high mean and variance-to-mean ratio.

It is recognized that this data from the ICP could be expected to be distributed as compound Poisson process and that the demand at each operational site could very well be Poisson distributed with an equal mean and variance. The

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[3]ASO personnel were unable to ascertain the number of operational sites to which this data pertained.

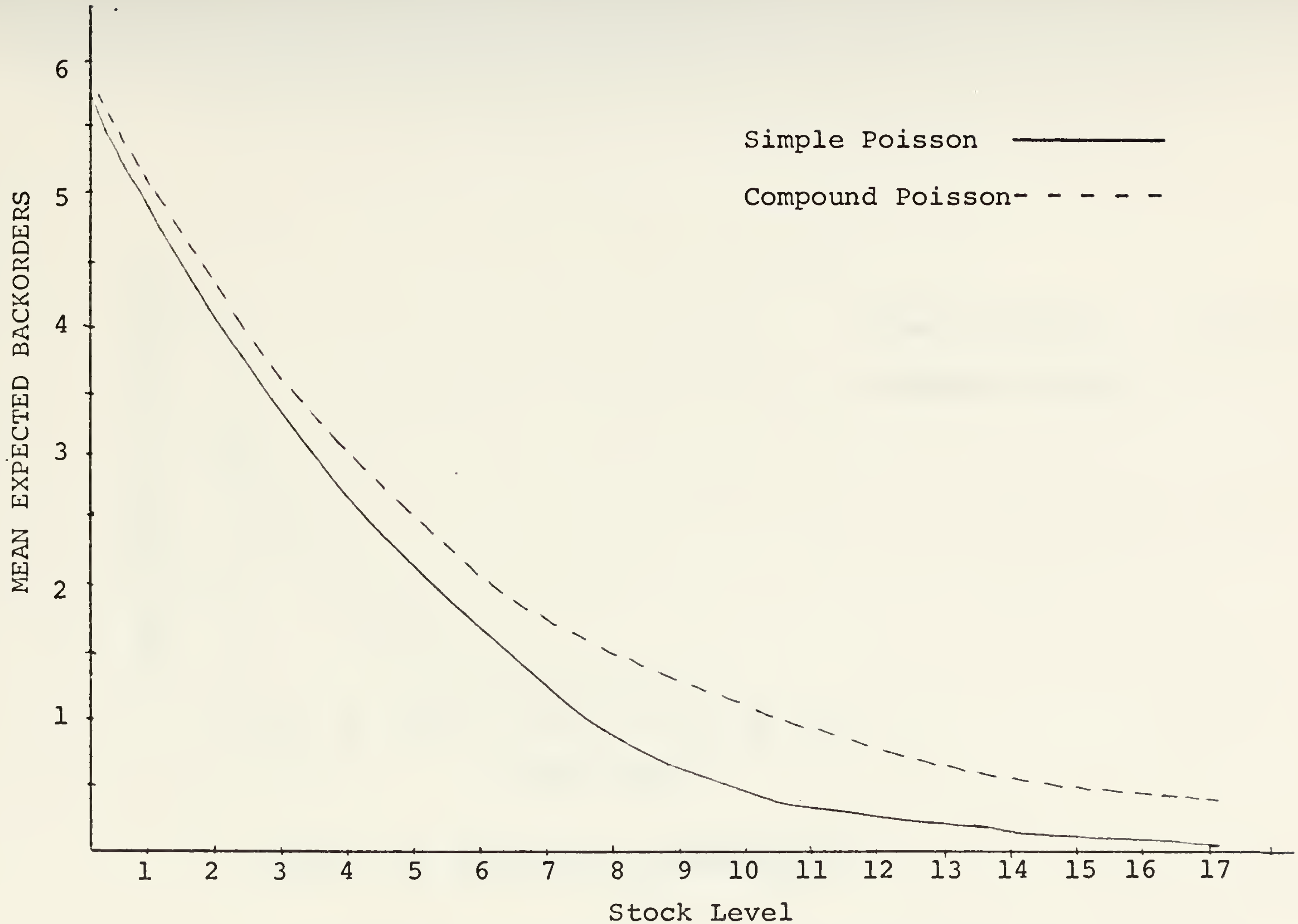

FIGURE 4.2 MEAN EBO VS. STOCK LEVEL (ALL ITEMS)

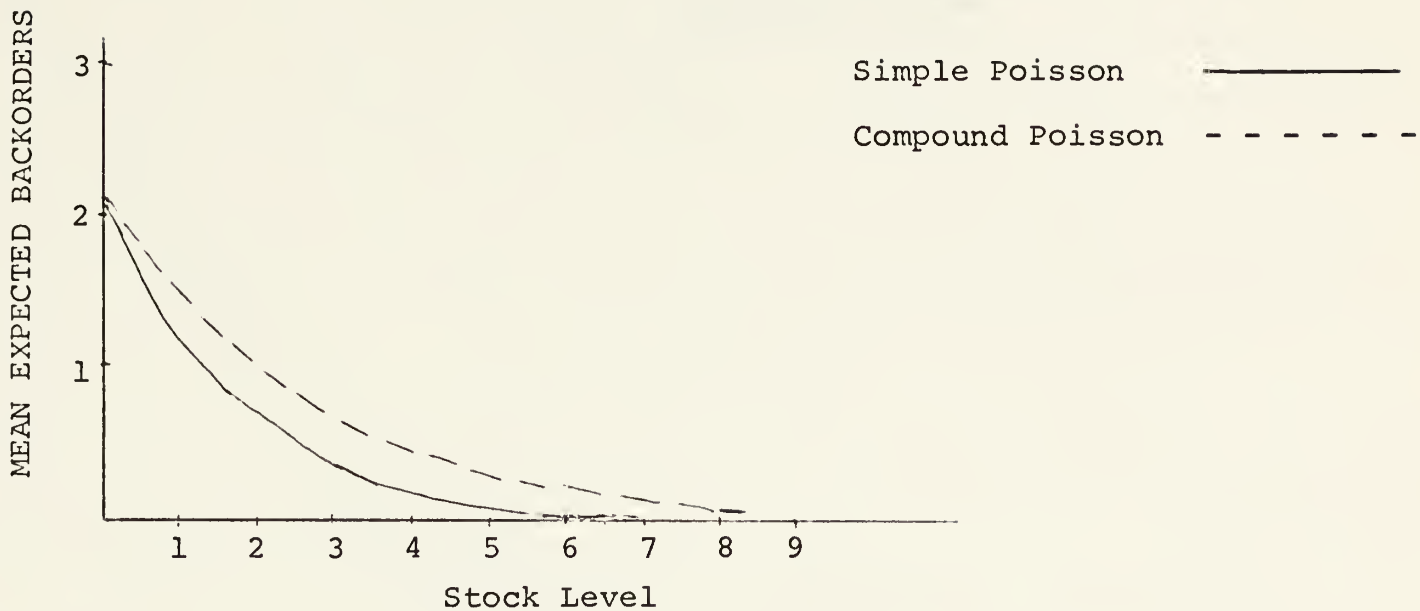

FIGURE 4.3 MEAN EBO VS CONSUMABLES

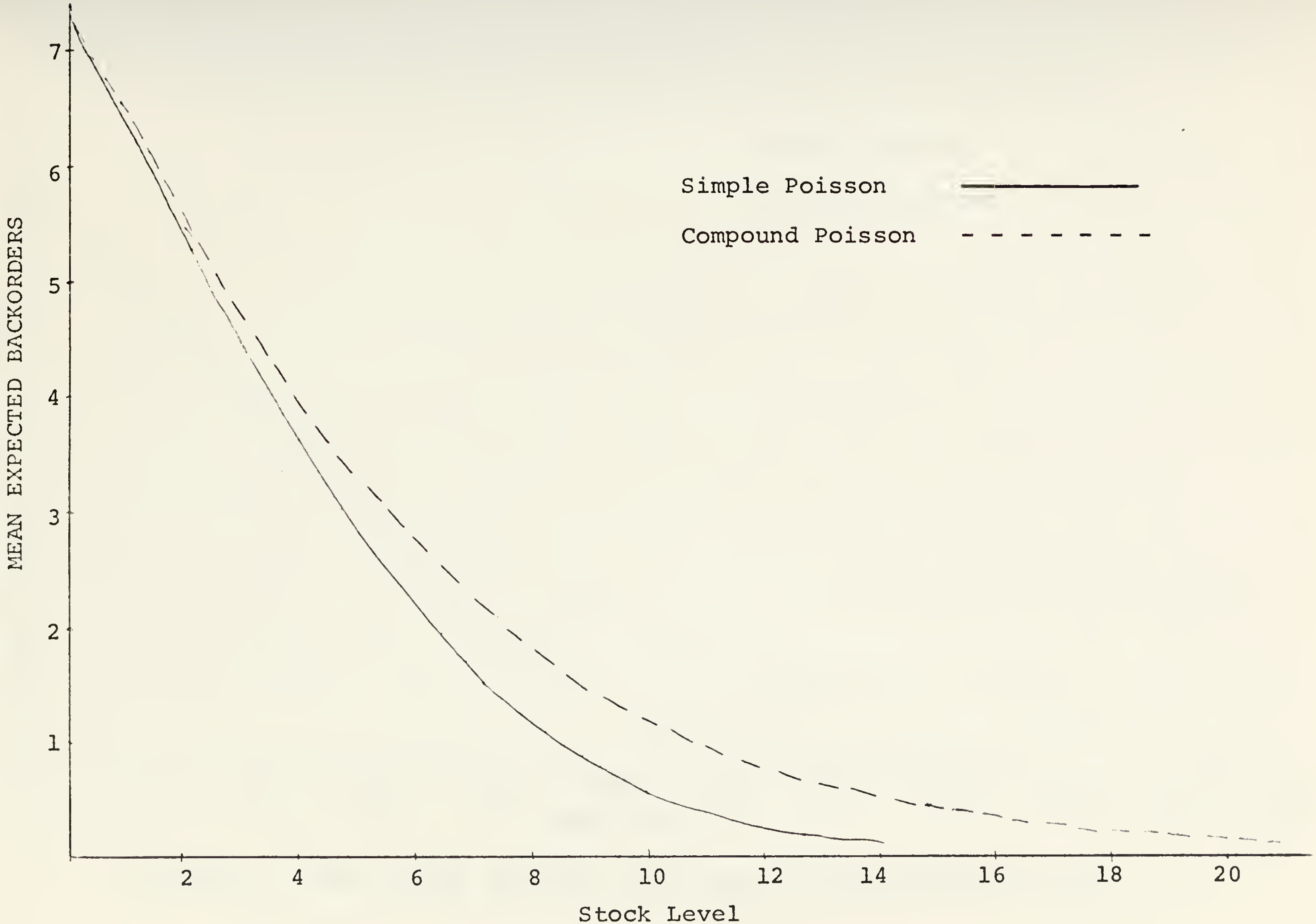

FIGURE 4.4 MEAN EBO VS. REPARABLES

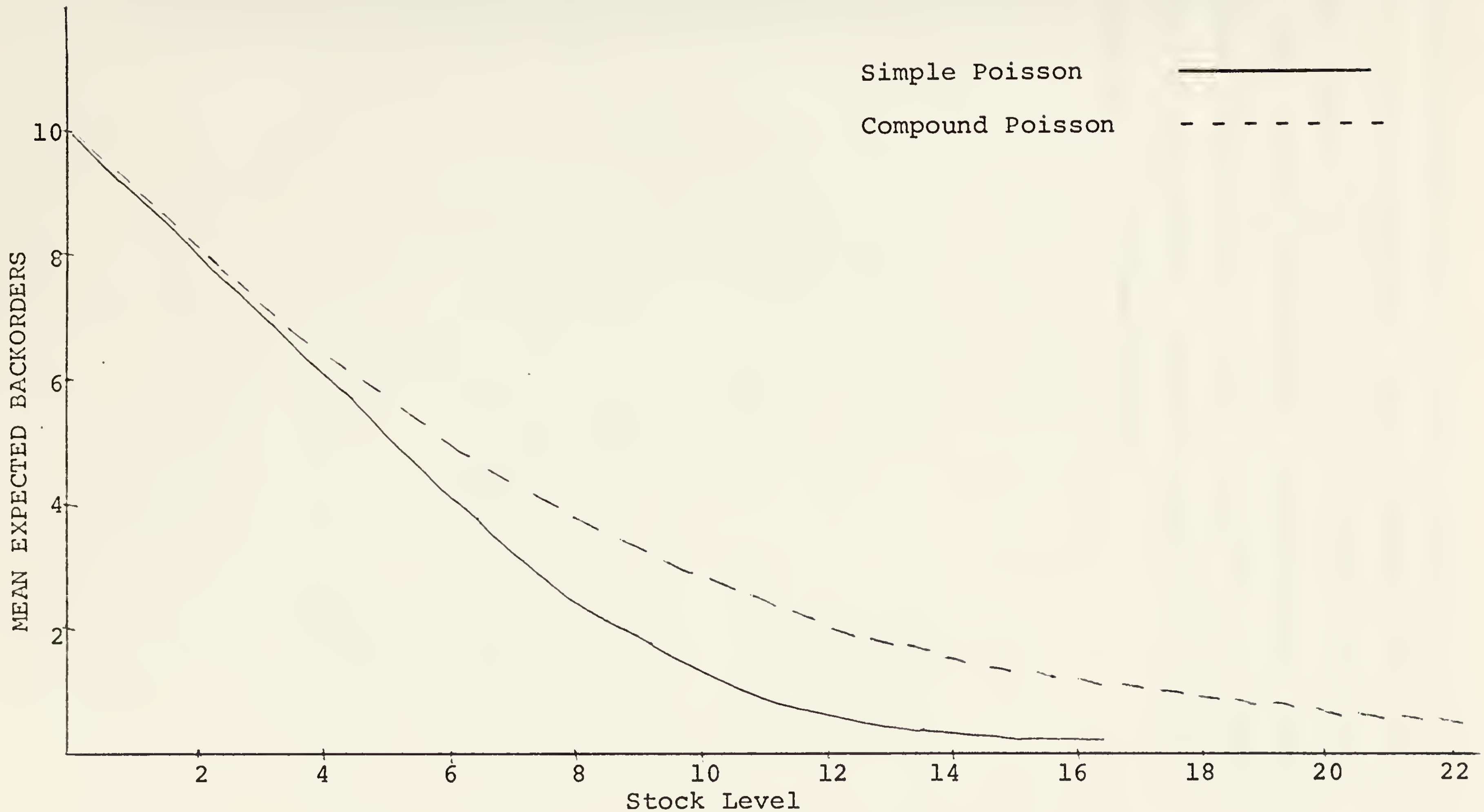

FIGURE 4.5 EBO VS ONE ITEM WITH HIGH MEAN AND VARIANCE/MEAN RATIO

figures indicate that if the mean and variance of demand are not equal at the unit level, then the expected backorders computed by the model are probably understated.

This chapter has discussed the Poisson and compound Poisson distributions and their properties and applications to the ASO IOL model. The next chapter will discuss conclusions and recommendations.

V. CONCLUSIONS AND RECOMMENDATIONS

This final chapter lists conclusions from this research and makes recommendations for improvement in the provisioning process.

A. CONCLUSIONS

1. Provisioning an aircraft weapons system is a complex project requiring careful, long leadtime planning with the most reliable data available.

2. The current ASO IOL model is a reasonable approach for optimization of an initial inventory selection on the basis of failure prediction, repair and resupply cycle times, and utilizing constraint options for funding, range, and depth of inventory.

3. Variation of demand data at the organizational level can have an effect on predicting expected backorders.

4. There is a conceptual conflict in the use of the Poisson distributions with its memoryless inter-event times and the IOL use of maintenance cycles.

5. Based upon the assumptions of the model, there are AWP rates which can be estimated from a marginal analysis approach to be

   a. a break-even rate, and
   b. a minimal investment cost rate.

6. Investment levels vary directly with increased resupply and repair times, but are more sensitive to the repair turn-around times.

7. Arbitrary decision rules, based on unit price and demand frequency, tend to eliminate lower cost, lower indenture items. To achieve comparable AWP rates (before exclusion), more higher cost, higher indenture components (which may also be bulkier) are retained, resulting in higher inventory investment.

B. RECOMMENDATIONS

1. The ASO IOL model should be verified at operational sites to determine if

a. predicted AWP rates are being realized,

b. demand variability affects the protections afforded by the IOL inventory.

2. Further study should review the conceptual differences between the memoryless inter-event time property of the Poisson process and the use of the flight-hour related maintenance cycles.

3. Sensitivity analysis should be conducted as early as possible in the provisioning cycle in order to determine an optimal AWP target and to determine if any additional resources could be diverted to the maintenance effort at the IMA level.

APPENDIX A

EXAMPLE OF IOL INVENTORY SELECTION

Program Data

RTAT	3 days
RST	90 days
MC	36

Component Data

Part #	9	9079	9079001	9079005
Cog	8R	2R	9N	9N
Cost	$9060	$1,130	$6.92	$40.80
MRF	.001	0	.3	.2
RPF	.074	.015	0	0
Inventory	0	0	0	1

9 (WRA)

$M = .1248$

$M_{WD} = \cancel{18.143}$

$EBO = \cancel{18.143}$

9079 (SRA)

$M = .018$

$M_{WD} = .018 + \cancel{18}$

$EBO = 18.018$

9079001 (PP)

$M = 10.8$

$EBO = 10.8$

9079005 (PP)

$M = 7.2$

$EBO = \cancel{7.2}$

4) $C/E = \dfrac{40.80}{.9995} = 40.8204$

$\Delta EBO = 18.143 - 17.1435$

3) $M_{WD}' = .1248 + 17.0187 = .9995$

$EBO' = 17.1435$

2) $M_{WD}' = .018 + 17.007 = 17.0187$

$EBO' = 17.0187$

1) $EBO' = 6.2007$

Other C/E ratios

09	9061.81
7905	1129.7
72001	6.919

<u>Procedure</u>

1. Add one 9N part 9079005. The EBO is reduced to 6.2007.

2. The expected number of SRA 9079 in the pipeline. M_{WD} changes to 17.0187, as do the EBO's since this part still has a zero stock level.

3. Updating the M_{WD} for WRA 09 reduces the EBO for the WRA to 17.1435, a change of .9995.

4. Compute the C/E ratio.

5. Repeat steps 1 through 4. The lowest C/E ratio is 6.919 for piece part 9079001. That item is added permanently to the inventory since it is well below the backorder penalty cost, which is normally in the vicinity of $1,000,000.

APPENDIX B DEMAND DATA AND STATISTICS OF AVIATION ITEMS (ARN 84 and APX 72)

TABLE B.1

COG	NIIN	QUARTERLY DEMAND 1	2	3	4	5	6	7	8	λ	VAR	q = VAR/λ
2R	1458304	5	11	7	6	7	9	0	2	5.9	12.7	2.2
8R	1462276	7	13	14	8	2	26	1	0	8.9	75.6	8.5
8R	1462277	1	7	6	11	5	20	7	8	8.1	31.0	3.8
1R	1473019	4	2	1	1	5	2	3	0	2.3	2.8	1.2
2R	1476073	4	11	5	8	11	8	0	4	6.4	14.6	2.3
2R	1476074	10	24	10	8	7	11	0	9	9.9	44.4	4.5
2R	1525095	7	15	12	20	18	17	3	7	12.4	37.7	3.0
1R	1530304	1	7	2	2	4	7	1	0	3.0	7.4	2.5
1R	1530305	0	4	1	0	3	4	2	0	1.8	3.1	1.7
1R	1530306	0	4	2	1	6	7	0	0	2.5	8.0	3.2
1R	1530307	0	4	1	1	1	4	1	1	1.6	2.3	1.4
2R	1531338	10	20	13	12	3	7	6	4	9.4	31.4	3.3
2R	1554712	10	16	10	11	5	6	0	1	7.4	29.1	3.9
2R	1631981	10	13	8	8	2	6	0	1	6.0	21.4	3.6
1R	1668166	1	1	1	2	3	5	0	4	2.1	3.0	1.4
1R	1674919	1	2	1	2	0	2	5	0	1.6	2.6	1.6
8R	1688769	10	24	40	39	42	77	16	14	32.8	480.2	14.6
8R	1688770	0	2	5	0	8	7	6	16	5.5	27.4	5.0
2R	1481141	1	4	1	0	1	2	0	2	1.4	1.7	1.2
2R	1525089	11	22	18	26	6	13	2	8	13.3	67.6	5.1
2R	1525090	10	15	6	4	5	2	5	1	6.0	20.6	3.4
2R	1525091	11	19	7	6	4	11	3	4	8.1	28.7	3.5
2R	1525092	14	8	5	11	13	2	3	2	7.3	24.5	3.4
2R	1525094	7	19	10	13	5	8	4	10	9.5	23.1	2.4
1N	081140	2	0	1	5	3	1	4	1	2.1	3.0	1.4
1R	0898010	30	15	31	7	13	15	11	11	16.6	80.0	4.8
2R	0898015	91	82	116	104	76	62	86	72	86.1	305.3	3.6
1R	2530867	49	52	43	24	27	40	22	15	34.0	188.6	5.6
1R	2530868	2	2	0	1	0	0	0	1	.8	.8	1.0
1R	4216531	0	3	0	0	0	0	0	0	.4	1.1	2.8
1R	1386328	25	40	9	16	17	7	17	7	17.25	122.5	7.1

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